

Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

- 1** Solve the equation $\ln(x+5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 2** Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 3 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - i| \leq 3$ and $|z| \geq |z - 4i|$. [4]

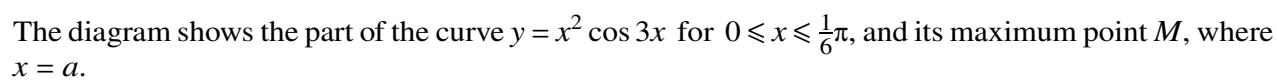
4 The parametric equations of a curve are

$$x = \frac{\cos \theta}{2 - \sin \theta}, \quad y = \theta + 2 \cos \theta.$$

Show that $\frac{dy}{dx} = (2 - \sin \theta)^2$.

[5]

[illegible]



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- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

- (b)** Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 6 (a)** Express $3 \cos x + 2 \cos(x - 60^\circ)$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [4]

[illegible]

- 7 (a) Use the substitution $u = \cos x$ to show that

$$\int_0^\pi \sin 2x e^{2\cos x} dx = \int_{-1}^1 2ue^{2u} du. \quad [4]$$

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8 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$.

[8]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 9** The lines l and m have equations

$$l: \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$

$$m: \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin O , the position vector of the point P is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

- (a) Given that l is perpendicular to m and that P lies on l , find the values of the constants a , b and c . [4]

[illegible]

- (b) The perpendicular from P meets line m at Q . The point R lies on PQ extended, with $PQ : QR = 2 : 3$.

Find the position vector of R .

[6]

[illegible]

10 Let $f(x) = \frac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 11** The complex number z is defined by $z = \frac{5a - 2i}{3 + ai}$, where a is an integer. It is given that $\arg z = -\frac{1}{4}\pi$.
- (a)** Find the value of a and hence express z in the form $x + iy$, where x and y are real. [6]

[illegible]

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- (b) Express z^3 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the simplified exact values of r and θ . [3]

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